February 1, 2008

LBL-36516
UCB-PTH-94/34

The Minimal Supersymmetric Model of Higgs-Higgs Condensation *

Yi-Yen Wu

Theoretical Physics Group
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

Abstract

This paper is motivated by three issues associated with the supersymmetric extension of the standard model: the μ problem, the possibility of raising the upper bound on the lightest-Higgs-boson mass, and the triviality problem associated with the Higgs sector. A new scheme based on the Higgs-Higgs condensation at low energy is proposed, and it is shown that these three issues are well solved by this scheme. As the first realization of this new scheme, the Minimal Supersymmetric Model of Higgs-Higgs Condensation (MSMHHC) is constructed and studied in detail. The MSMHHC is identical with the MSSM (Minimal Supersymmetric Standard Model) in the fundamental particle content, and their lagrangians differ only in the Higgs sector. The Higgs sector of the MSMHHC is based on the softly-broken supersymmetric Nambu−Jona-Lasinio model with the four-field interaction of the Higgs doublets. At low energy, these two Higgs doublets condense into two neutral Higgs gauge singlets, and the low-energy effective lagrangian of the MSMHHC has the form of the non-minimal supersymmetric standard model which contains two more Higgs singlets than the MSSM. Another unique feature of the MSMHHC is that heavy top quark always implies strongly-interacting low-energy Higgs sector, i.e., a large mass for the lightest Higgs boson. A systematic study of the parameter space is also made in order to reveal the qualitative features of the MSMHHC. Finally, we comment on the question *"How large can the lightest-Higgs mass be?"* with the conclusion that the supersymmetric model of Higgs-Higgs condensation will be the most promising candidate if the lightest-Higgs mass of the MSSM is excluded by the future experiments.

Disclaimer

1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) is by far the most studied supersymmetric extension of the standard model. However, there are still unsolved issues about the MSSM, and it may be necessary to go beyond the MSSM for a better solution. In this paper, we will confine ourselves to the issues of the Higgs sector. The first issue is the μ-term problem, $\mu(H_1\epsilon H_2)$, of the MSSM [1, 2], where H_1 and H_2 are the two Higgs-doublet superfields. In the MSSM, we need μ to be non-zero and of order the weak scale. However, since $\mu(H_1\epsilon H_2)$ is supersymmetric and gauge invariant, there is no reason for μ not to be of order the Planck scale or the GUT scale. Thus, there is really a naturalness problem of μ in the MSSM and ideas beyond the MSSM are needed to solve this problem in a natural way. The second issue is about the upper bound on the lightest-Higgs-boson mass. It is well-known that, in the MSSM, the lightest-Higgs mass at tree level cannot get larger than the mass of gauge boson Z, and this upper bound is raised as much as 20 (50) GeV for a top-quark mass of 150 (200) GeV [3, 4]. However, there is no guarantee of finding a light Higgs below the upper bound of the MSSM in future experiments. It is then of essential importance to go beyond the MSSM and consider those models which can raise the upper bound of the lightest-Higgs mass above the prediction of the MSSM naturally. One of those existing models is the extension of the MSSM by including more Higgs gauge singlets in the Higgs sector and more non-trivial Higgs self-interactions in the superpotential (i.e., the Non-Minimal Supersymmetric Standard Model, Non-Minimal SSM [5]). In the Non-Minimal SSM, a large mass for the lightest Higgs boson always implies a strongly-interacting Higgs sector. And, a strongly-interacting Higgs sector suffers from the triviality problem associated with the Higgs self-couplings [6, 7, 8]. Therefore, the third issue is how this triviality problem can be resolved at high energy. The above three issues actually constitute the main motivations of this paper, and it turns out that the Minimal Supersymmetric Model of Higgs-Higgs Condensation (MSMHHC) proposed here does provide the natural answers to all these issues. Next, we will briefly describe the existing approaches and our approach to these issues.

Let's begin with the first issue, the μ problem. There have been two at-

titudes towards the natural μ-term: either derive the μ-term directly from the high-energy fundamental theory (e.g., the superstring or supergravity) or generate the μ-term within the low-energy (i.e., lower than the Planck scale) effective models. The latter attitude is adopted in this paper and, since those works which adopt the former attitude are irrelevant to our discussions of the second and the third issues, the attention will be focused only on those works which adopt the latter attitude. A direct step beyond the MSSM is the Next to Minimal Supersymmetric Standard Model (NMSSM or (M+1)SSM) [9], which has one Higgs gauge singlet N and two Higgs doublets H_1, H_2. The relevant piece in the superpotential W_{NMSSM} is $hN(H_1 \epsilon H_2)$, where h is the Higgs coupling. In the NMSSM, the μ-term is effectively generated through $\mu = h < \phi_N >$, where ϕ_N is the scalar component of N. However, the naturalness problem of μ remains unsolved in the NMSSM because there is again the naturalness problem of $< \phi_N >$. Another effective approach is the Minimal Top-Condensate Supersymmetric Standard Model (MTCSSM) [10, 11, 12, 13], where there is no fundamental Higgs field and the Higgs at low energy is actually the top condensate. Starting with the Supersymmetric Nambu−Jona-Lasinio (SUSY NJL) model with the four-field interaction of the top defined at the cut-off, the low-energy effective lagrangian of the MTCSSM always contains the μ-term. It was pointed out in [12] that μ is naturally of order the soft SUSY-breaking scalar mass. Hence, μ is naturally small and the MTCSSM does solve the μ problem.

Our approach, the Minimal Supersymmetric Model of Higgs-Higgs Condensation (MSMHHC), which has exactly the same fundamental particle content as the MSSM does, is based on the softly-broken SUSY NJL model with the four-field interaction of two Higgs doublets, H_1 and H_2, defined at the cut-off Λ. At low energy, H_1 and H_2 condense into two composite neutral Higgs-singlet superfields, M and N [12], and the superpotential W_{MSMHHC} of the low-energy effective lagrangian has the form of the Non-Minimal SSM: $W_{MSMHHC} = mMN + hN(H_1 \epsilon H_2)$ (the soft SUSY-breaking terms are not written down here), where m and h are coupling constants. So, the MSMHHC looks like the MSSM at high energy, but behaves like the Non-Minimal SSM with two Higgs singlets (which we call the (M+2)SSM) at low energy. Obviously, the MSMHHC generates the μ-term in the same way as the NMSSM does, where $\mu = h < \phi_N >$. Nevertheless, unlike the NMSSM, $< \phi_N >$ is generated dynam-

ically in the MSMHHC due to supersymmetry breaking and naturally of order the soft SUSY-breaking scalar mass. That is, $< \phi_N >$ vanishes in the supersymmetric limit. Therefore, μ is naturally small because $h \approx \mathrm{O}(1)$ naturally in the MSMHHC. The MSMHHC does provide a natural solution to the μ problem. This is indeed an explicit realization of the suggestion in [2]: μ is zero initially, and a non-zero value is induced only through supersymmetry breaking effects.

In fact, there will be another potential disadvantage of the NMSSM (or, any Non-Minimal SSM containing fundamental neutral Higgs singlets) if the fundamental Higgs singlet is introduced to generate the μ-term. Assuming certain SUSY grand unification schemes, the Higgs singlet coupled to the Higgs doublets at low energy may destroy the hierarchy between the light fields and the superheavy fields if the Higgs singlet is also coupled to the superheavy fields [14]. However, in the MSMHHC, there is no fundamental Higgs singlet and the Higgs singlets at low energy are composite. Therefore, in the MSMHHC, there is no danger of destroying the hierarchy between the light and the superheavy fields. From this viewpoint, the MSMHHC shares the advantages of both the MSSM at high energy and the Non-Minimal SSM at low energy, but avoids the disadvantages of both the MSSM at low energy and the Non-Minimal SSM at high energy. Even as a purely theoretical construction, the MSMHHC is worth studying because it provides us with a interesting dynamic connection between the MSSM at high energy and the Non-Minimal SSM at low energy.

For the sake of completeness, we also mention those works on the μ problem in the context of supergravity or superstring theories briefly, although they are not really relevant to our approach. Based on Peccei-Quinn symmetry [15], the μ problem and the strong-CP problem can be combined, and the μ-term can be generated by a composite axion in the hidden sector [16, 17] or by a generalized Higgs mass term [18] naturally. There are other approaches without Peccei-Quinn symmetry [19]. Notice that [16, 17] is the first work that proposed the idea of compositeness in solving the μ problem. However, in [16, 17], the μ-term is generated by coupling the hidden-sector superfields S_1, S_2 to the Higgs superfields as $\frac{1}{M} S_1 S_2 (H_1 \epsilon H_2)$ ($M \simeq 2.43 \times 10^{18}$ GeV), and the scalar components of S_1, S_2 condense into the composite axion in the hidden sector. Hence, the scheme and the main results of our model MSMHHC, which is based on the Higgs-Higgs condensation, are completely different from those of [16, 17].

Now, turn to the second and the third issues. In general, in the Non-Minimal SSM, the upper bound on the lightest-Higgs mass is expected to be relaxed because the tree-level constraint on the lightest-Higgs mass of the MSSM is no longer valid. For example, in the NMSSM, $m_{LH} \le \frac{1}{\sqrt{2}} hv$ (when $\tan\beta$=1) has been derived in [7], where $v \approx$250 GeV. m_{LH} denotes the physical mass of the lightest Higgs boson throughout this paper. Increasing the Higgs coupling h therefore increases the upper bound. However, as mentioned before, one then has to face the triviality problem. That is, when scaled toward high energy, h will blow up faster if a larger value of h is assumed at low energy. Based on the observation of triviality, several estimates of the upper bound of m_{LH} in the NMSSM have been made [7, 8, 20], and they are indeed larger than that of MSSM. To go beyond the triviality problem, it has been argued in [21] that the problem of triviality is possibly an indication of compositeness. From this viewpoint, the MSMHHC is precisely the realization of the argument of [21] applied to the triviality problem in the Non-Minimal SSM. In the MSMHHC, the composite field N is static at tree level, but develops a kinetic term at low energy, $Z_N \int d^4\theta\, N^\dagger N$, from quantum corrections [11], where $Z_N \sim \frac{1}{8\pi^2}\ln(\frac{\Lambda^2}{\mu_E^2})$ is the wave-function renormalization and μ_E is the renormalization scale. The superpotential of the low-energy effective lagrangian is $W_{MSMHHC} = mMN + hN(H_1\epsilon H_2)$, and $h \sim \frac{1}{\sqrt{Z_N}}$ because we have rescaled N ($N \to \frac{1}{\sqrt{Z_N}}N$) in order to have the correct normalization of the kinetic term in the low-energy effective lagrangian. In the limit $\mu_E \to \Lambda$, the kinetic term of N vanishes (i.e., N is then static) and h blows up. Therefore, the triviality problem is resolved within the MSMHHC in the sense that the triviality problem seems to arise only because N is mistaken for a dynamical field near the cut-off Λ. The MSMHHC, which looks like the Non-Minimal SSM at low energy but without the plague of triviality, is a promising answer to the last two issues. In Sections 5−6, it will further be illustrated that the large value of m_{LH} predicted by the MSMHHC is closely related to the strong top Yukawa interaction.

The MTCSSM is irrelevant to the above discussion because the low-energy form of the MTCSSM is simply the MSSM and there is no Higgs self-interaction. Although the MTCSSM can also solve the μ problem, the MSMHHC solves it with a scheme different from that of the MTCSSM, and therefore is worth studying. Besides, the MSMHHC (whose low-energy theory is the (M+2)SSM) is more interesting than the MTCSSM (whose low-energy theory is the MSSM)

from the viewpoint of raising the upper bound on the lightest Higgs mass. The present version of the MSMHHC contains not only the Higgs sector but also the top Yukawa term. It is reasonable to neglect the other quark and lepton Yukawa couplings for a qualitative description of the phenomena under study. The bottom Yukawa coupling is also neglected because the possibility of $\tan\beta \gg 1$ is not considered in this paper. In the following, we shall briefly describe the organization of this paper.

In Section 2, the MSMHHC is defined and its low-energy effective lagrangian is derived using the SUSY NJL technique. The dynamics of the composite Higgs singlets is discussed, and the triviality problem is solved naturally. In Section 3, in order to study the MSMHHC vacuum and related physical quantities, the effective potential of the MSMHHC is computed. In Section 4, from the minimization of the effective potential, the vacuum constraint equations are derived and the MSMHHC vacuum is examined. In the MSMHHC, the spontaneous breaking of electroweak symmetry accompanies the condensation of Higgs doublets, and therefore, compared to the conventional NJL model, the MSMHHC has two more vacuum constraints. In Section 5, the full mass spectrum of the MSMHHC is analyzed. A unique feature of the MSMHHC is that the lightest Higgs boson will be massless if the top Yukawa interaction is turned off, and the lightest Higgs boson becomes massive only through the effects of the top Yukawa interaction. It is then shown that, in the MSMHHC, strong top Yukawa interaction (i.e., heavy top quark) usually implies a large mass for the lightest Higgs boson. The same conclusion will also be reached elsewhere in this paper.

In Section 6, to study the phenomenological aspects of the MSMHHC in general, its full parameter space is defined, and its behavior over the parameter space is examined in Sections 6−9. All the physical quantities are first computed from the effective potential at the cut-off scale, and then renormalized down to low energy properly. Only two phenomenologically reasonable constraints are assumed in the analysis of the parameter space. The first constraint requires that all the soft SUSY-breaking scalar squared masses in the Higgs and the top sectors be non-negative at low energy, and this leads to two non-trivial results. First, this constraint implies a simple relationship among the soft SUSY-breaking scalar masses. Second, under this constraint, strong top Yukawa interaction

always implies strongly-interacting low-energy Higgs sector (i.e., large m_{LH}). The conclusion of Section 5 is re-established in a different way, and therefore it is indeed a consistent feature of MSMHHC. The second constraint requires the physical top-quark mass to be $164 \sim 180$ GeV, and, together with the first constraint, it implies that the MSMHHC should be an effective intermediate-scale model, i.e., weak scale $\ll \Lambda \ll$ Planck scale. However, there is a fine-tuning problem associated with the soft SUSY-breaking parameters (although it's not severe). It is argued that this fine-tuning problem is actually a guide to the future model-building, not a real obstacle. In Sections 7−9, the low-energy physical quantities, such as the mass of the lightest Higgs m_{LH}, $\tan\beta_r$, μ_r, etc., are computed and their dependence on the parameter space is examined systematically. In Section 7, the dependence on the cut-off Λ is studied. In one example, the MSMHHC predicts 150 GeV $< m_{LH} <$ 400 GeV. The fact that m_{LH} can be as large as 400 GeV indeed solves the second issue. In Section 8, the dependence on F_{SUSY} (the strength of soft SUSY breaking) is studied. As an explicit answer to the μ problem, the dependence of the effective μ_r on F_{SUSY} is emphasized. Another important phenomenon is the saturation of m_{LH} in the limit of large F_{SUSY}. In Section 9, the dependence on the soft SUSY-breaking pattern is studied, and we are especially interested in how $\tan\beta_r$ and m_{LH} depend on the soft SUSY-breaking pattern. Finally, it is pointed out in Section 10 that, in order to make the phenomenological study of the MSMHHC more complete, direct extensions of the present model are necessary and guidelines for the future model-building are needed. We also comment on the interesting question: *"How large can the lightest-Higgs mass be?"*

2 The Minimal Supersymmetric Model of Higgs-Higgs Condensation

The Minimal Supersymmetric Model of Higgs-Higgs Condensation (MSMHHC) is minimal in the sense that the MSMHHC and the MSSM are identical in the fundamental particle content, and their lagrangians differ only in the Higgs sector. Viewing this work as a first attempt at the idea of Higgs-Higgs condensation, we choose to study the main physical features of the MSMHHC rather than make it a phenomenologically complete model. Therefore, reasonable sim-

plifications will be made whenever it is necessary. We shall ignore all quark and lepton Yukawa couplings except the one associated with the top quark, since the others are inessential to the qualitative description of the phenomena under study. Notice that the bottom Yukawa coupling is also neglected because the possibility of $\tan\beta \gg 1$ is not considered in this paper. The $SU(3)_c$ color symmetry is ignored because we are mainly concerned with the scalar Higgs sector. We then start with the following lagrangian of the MSMHHC, $\mathcal{L}_\Lambda$, which is a non-renormalizable model with momentum cut-off Λ:

$$
\begin{aligned}
\mathcal{L}_\Lambda \;=\; & \int d^4\theta \,\{\, H_1^\dagger H_1(1-m_1^2\theta^2\bar\theta^2) + H_2^\dagger H_2(1-m_2^2\theta^2\bar\theta^2) \\
& \qquad\qquad +(Q^\dagger Q + T_C^\dagger T_C)(1-m_T^2\theta^2\bar\theta^2)\,\} \\
& + \int d^4\theta\, G(H_1\epsilon H_2)^\dagger (H_1\epsilon H_2)[1+B\theta^2+B\bar\theta^2+(B^2-m_M^2)\theta^2\bar\theta^2] \\
& + \int d^2\theta\,\{m_{12}^2(H_1\epsilon H_2)\theta^2 + f_T(H_2\epsilon Q)T_C\} \\
& + \int d^2\bar\theta\,\{m_{12}^2(H_1\epsilon H_2)^\dagger\bar\theta^2 + f_T(H_2\epsilon Q)^\dagger T_C^\dagger\}
\end{aligned}
\tag{1}
$$

The MSMHHC has the global symmetry SU(2)×U(1), and its Higgs sector is based on the softly-broken SUSY NJL model. H_1 and H_2 are the two SU(2) Higgs doublets. Q is the SU(2) doublet of the top and the bottom chiral superfields. T_C is the SU(2) singlet of the top (the SU(2) bottom singlet is omitted since the bottom Yukawa coupling is neglected). ϵ is the usual 2×2 antisymmetric ϵ-tensor, and $(H_1\epsilon H_2) = \epsilon_{ij}H_{1i}H_{2j}$ is implied. The convention for superspace notations of [22] is adopted. f_T is the top Yukawa coupling. G is the four-Fermi coupling constant of dimension mass^{-2}. m_1, m_2, m_T, m_M, B and m_{12} are the six soft SUSY-breaking parameters of dimension mass.

To generalize the usual NJL technique to the supersymmetric case, it was first pointed out in [13, 23] that one has to introduce two chiral superfields (denoted as M and N here) in order to write a linearized version of the SUSY NJL model. Therefore, with the introduction of M and N, $\mathcal{L}_\Lambda$ can be written in a more instructive form:

$$
\begin{aligned}
\mathcal{L}_\Lambda \;=\; & \int d^4\theta \,\{\, H_1^\dagger H_1(1-m_1^2\theta^2\bar\theta^2) + H_2^\dagger H_2(1-m_2^2\theta^2\bar\theta^2) \\
& \qquad +M^\dagger M(1-m_M^2\theta^2\bar\theta^2) + (Q^\dagger Q + T_C^\dagger T_C)(1-m_T^2\theta^2\bar\theta^2)\,\}
\end{aligned}
$$

$$+\int d^2\theta\,\{m_{12}^2(H_1\epsilon H_2)\theta^2 + mMN(1-B\theta^2) + hN(H_1\epsilon H_2) + f_T(H_2\epsilon Q)T_C\}$$
$$+\int d^2\bar\theta\,\{m_{12}^2(H_1\epsilon H_2)^\dagger\bar\theta^2 + mM^\dagger N^\dagger(1-B\bar\theta^2) + hN^\dagger(H_1\epsilon H_2)^\dagger + f_T(H_2\epsilon Q)^\dagger T_C^\dagger\} \tag{2}$$

where

$$G = \frac{h^2}{m^2} \tag{3}$$

and, from (2), the Euler-Lagrange equations of M and N are:

$$\begin{aligned} M &= -\frac{h}{m}(H_1\epsilon H_2)(1+B\theta^2) \\ N &= \frac{h}{4m^2}(1+B\theta^2)\{\bar D^2[(H_1\epsilon H_2)^\dagger(1+B\bar\theta^2 - m_M^2\theta^2\bar\theta^2)]\} \end{aligned} \tag{4}$$

It is clear from (4) that M and N are indeed neutral SU(2) singlet composite chiral superfields. By substituting (4) into (2), (2) is equivalent to the original lagrangian (1). Before discussing the low-energy effective lagrangian for (2), let's explain the choice of soft SUSY-breaking parameters. The set of these six soft SUSY-breaking parameters, (m_1, m_2, m_T, m_M, B, m_{12}), is actually the minimal choice from the viewpoint of dynamical chiral symmetry breaking and the consideration of the global SU(2)×U(1) symmetry. It was pointed out in [10, 11, 13, 23] that soft SUSY-breaking terms are necessary in order to get a chiral symmetry breaking vacuum and to induce condensation. (m_1, m_2, m_T, m_M) turns out to be the minimal choice required by the MSMHHC from this viewpoint. However, without (B, m_{12}), the Higgs sector of $\mathcal{L}_\Lambda$ has an SU(2)×U(1)×U(1) symmetry. Therefore, (B, m_{12}) is indeed the minimal choice that can explicitly break this symmetry to the correct electroweak SU(2)×U(1) symmetry. In general, we could have chosen non-universal soft SUSY-breaking scalar squared masses at the cut-off for Q and T_C in (1). However, this issue of non-universality is not essential to our main concerns here.

In (2), $\mathcal{L}_\Lambda$ looks like the Non-Minimal SSM which has two more Higgs gauge singlets, M and N, than the MSSM does. However, at tree level, N remains static. When quantum corrections are included, N does develop a kinetic term [10, 11]. According to (2), there are four divergent (when $\Lambda \to \infty$) supergraphs at one loop, and their contributions can be computed easily:

$$\Sigma_N \int d^4\theta\, N^\dagger N[1 + (m_1^2+m_2^2)\theta^2\bar\theta^2],$$

$$
\begin{aligned}
&\Sigma_{H_2} \int d^4\theta \, H_2^\dagger H_2[1 + 2m_T^2\theta^2\bar{\theta}^2], \\
&\Sigma_Q \int d^4\theta \, Q^\dagger Q[1 + (m_2^2 + m_T^2)\theta^2\bar{\theta}^2], \\
&\Sigma_{T_C} \int d^4\theta \, T_C^\dagger T_C[1 + (m_2^2 + m_T^2)\theta^2\bar{\theta}^2]
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\Sigma_N &= N_W \frac{h^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}), \\
\Sigma_{H_2} &= \Sigma_Q = \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}), \\
\Sigma_{T_C} &= N_W \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2})
\end{aligned} \tag{6}
$$

where μ_E is the renormalization scale, and N_W is the dimension of the SU(2) representation. In the present case, N_W=2. The first term in (5), which corresponds to the supergraph of Fig.1, is indeed the kinetic term of N. Similar to Fig.1, the other terms in (5) are generated through the top Yukawa interaction. The wave-function renormalization constants for these superfields, in the one-loop approximation, are then defined as follows:

$$
\begin{aligned}
Z_N &= \Sigma_N, \qquad Z_{H_2} = 1 + \Sigma_{H_2}, \\
Z_Q &= 1 + \Sigma_Q, \qquad Z_{T_C} = 1 + \Sigma_{T_C}
\end{aligned} \tag{7}
$$

H_1 and M do not get renormalized. The above results show that these two composite Higgs singlets, M and N, are true dynamical degrees of freedom at low energy ($\mu_E \ll \Lambda$), that is, we should see two Higgs doublets and two Higgs singlets at low energy. However, as $\mu_E \to \Lambda$, $Z_N \to 0$ and therefore it no longer makes sense to treat N as a true dynamic superfield near the cut-off.

Up to some finite contributions, the low-energy effective lagrangian $\mathcal{L}_{eff}$ can be obtained by absorbing the wave-function renormalization constants and re-defining the coupling constants according to the results of (5) and (6):

$$
\mathcal{L}_{eff} =
$$

$$
\int d^4\theta \, \{ \, H_{1r}^\dagger H_{1r}(1 - m_{1r}^2\theta^2\bar{\theta}^2) + H_{2r}^\dagger H_{2r}(1 - m_{2r}^2\theta^2\bar{\theta}^2)
$$

$$
\begin{aligned}
&+M_r^\dagger M_r(1-m_{Mr}^2\theta^2\bar\theta^2)+N_r^\dagger N_r(1-m_{Nr}^2\theta^2\bar\theta^2) \\
&+Q_r^\dagger Q_r(1-m_{Qr}^2\theta^2\bar\theta^2)+T_{Cr}^\dagger T_{Cr}(1-m_{T_{Cr}}^2\theta^2\bar\theta^2)\,\} \\
&+\int d^2\theta\,\{m_{12r}^2(H_{1r}\epsilon H_{2r})\theta^2+m_rM_rN_r(1-B_r\theta^2)+h_rN_r(H_{1r}\epsilon H_{2r})+f_{Tr}(H_{2r}\epsilon Q_r)T_{Cr}\} \\
&+\int d^2\bar\theta\,\{m_{12r}^2(H_{1r}\epsilon H_{2r})^\dagger\bar\theta^2+m_rM_r^\dagger N_r^\dagger(1-B_r\bar\theta^2)+h_rN_r^\dagger(H_{1r}\epsilon H_{2r})^\dagger+f_{Tr}(H_{2r}\epsilon Q_r)^\dagger T_{Cr}^\dagger\}
\end{aligned}
\tag{8}
$$

where a subscript r is used to distinguish the fields and couplings defined at low energy from those defined at the cut-off Λ. The non-trivial relations between these two sets of fields and couplings are organized as follows:

$$N_r = \sqrt{Z_N}N,\ H_{2r}=\sqrt{Z_{H_2}}H_2,\ Q_r=\sqrt{Z_Q}Q,\ T_{Cr}=\sqrt{Z_{T_C}}T_C \tag{9}$$

$$
\begin{aligned}
m_{Nr}^2 &= -(m_1^2+m_2^2) \\
m_{2r}^2 &= \frac{m_2^2-2\Sigma_{H_2}m_T^2}{1+\Sigma_{H_2}} \\
m_{12r}^2 &= \frac{m_{12}^2}{\sqrt{Z_{H_2}}} \\
m_{Qr}^2 &= \frac{(1-\Sigma_Q)m_T^2-\Sigma_Q m_2^2}{1+\Sigma_Q} \\
m_{T_{Cr}}^2 &= \frac{(1-\Sigma_{T_C})m_T^2-\Sigma_{T_C} m_2^2}{1+\Sigma_{T_C}}
\end{aligned}
\tag{10}
$$

$$f_{Tr} = \frac{f_T}{\sqrt{Z_{H_2}Z_QZ_{T_C}}} \tag{11}$$

$$m_r = \frac{1}{\sqrt{G}}\cdot\frac{1}{\sqrt{\frac{N_W}{16\pi^2}\ln(\frac{\Lambda^2}{\mu_E^2})}} \tag{12}$$

$$h_r = \frac{1}{\sqrt{1+\Sigma_{H_2}}}\cdot\frac{1}{\sqrt{\frac{N_W}{16\pi^2}\ln(\frac{\Lambda^2}{\mu_E^2})}} \tag{13}$$

Those fields and couplings that do not get renormalized are not listed. Therefore, at low energy, the MSMHHC becomes the Non-Minimal SSM with the couplings defined above. This Non-Minimal SSM will be called the (M+2)SSM. Notice

that these low-energy couplings depend only on G, not on h or m. Relations (10)−(13) will be useful later.

From (13), the physical meaning of the low-energy Higgs self-coupling h_r is clear: it simply reflects the cut-off dependence of $\mathcal{L}_{eff}$. In the limit $\mu_E \to \Lambda$, $Z_N \to 0$ and the behavior of triviality is reproduced: $h_r \to \infty$. Therefore, the issue of triviality is resolved in the sense that the problem of triviality seems to arise only because the composite N is mistaken for a fundamental dynamical field near the cut-off Λ. Of course, the complete resolution of triviality would require us to go beyond the cut-off Λ and to search for the correct renormalizable theory at higher energy from which the MSMHHC can be derived. However, this is beyond the scope of this paper.

In (10), $m^2_{Nr} = -(m^2_1 + m^2_2)$ is negative. The negative value of m^2_{Nr}, which is induced purely by the soft SUSY-breaking scalar squared masses, suggests that condensation should occur and ϕ_{N_r}, the scalar component of N_r, should develop a non-trivial VEV. This is consistent with the observation made in [10, 11, 13, 23] that, in the SUSY NJL model, there is no condensation in the supersymmetric limit and soft SUSY-breaking scalar-mass terms must be included to trigger condensation. (Details can be found in Section 4.) Therefore, the VEV $< \phi_{N_r} >$ is induced only through supersymmetry breaking effects and is naturally of order F_{SUSY}, the strength of soft SUSY-breaking scalar mass. This is exactly the solution to the naturalness problem of μ, where μ is effectively generated by $\mu_r = h_r < \phi_{N_r} >$ here. (h_r is also natural since its dependence on Λ is logarithmic.) Aside from the above argument, the computations of μ_r versus Λ and μ_r versus F_{SUSY} are done in Sections 7−8, which constitute the concrete answer to the μ problem.

In order to simplify the computations but keep track of the same physics at the same time, another simplification will be made in the following sections: the SU(2) symmetry is turned off and all the superfields are then singlets. That is, the MSMHHC of U(1) will be studied instead of the MSMHHC of SU(2)×U(1). Notice that the results of this section, (2)−(13), are still valid under this simplification except for N_W=1. Although the computations and figures presented in the following sections are obtained by assuming the MSMHHC of U(1), their physical features are shared by both the MSMHHC of U(1) and the MSMHHC of SU(2)×U(1). As we shall see later, the qualitative features of the MSMHHC

obtained under this simplification in this paper remain true even in the general SU(2)×U(1) case.

3 The Effective Potential

In order to study the phenomena of condensation, one must go beyond the tree-level computations of the SUSY NJL model. Therefore, the effective potential of the MSMHHC will be computed up to one loop, i.e., $V_{eff} = V_{tree} + V_{1-loop}$. Notice that we are now working with the MSMHHC of U(1) symmetry. Using the Coleman-Weinberg one-loop effective potential [24]:

$$V_{1-loop} = \frac{1}{2} STr \int \frac{d^4p}{(2\pi)^4} \ln(p^2 + \hat{M}^2) \tag{14}$$

where $STr(\hat{M}^2) = Tr(\hat{M}_B^2) - 2\,Tr(\hat{M}_F^2)$. The convention of [25] will be adopted in the computations of the spin-0 and spin-$\frac{1}{2}$ mass matrices, $\hat{M}_B$ and $\hat{M}_F$. Care should be taken in treating the superfield N since it is static at tree level. The convention of component notations for the chiral superfield N is $N = \phi_N + \sqrt{2}\theta\psi_N + \theta\theta F_N$, and it applies to other chiral superfields.

Notice that $m_{12}^2(H_1 \epsilon H_2)\theta^2$ in (2) can be written as $-\frac{m_{12}^2}{\sqrt{G}} M\theta^2$ with a shift in N. V_{tree} of the MSMHHC of U(1) symmetry can be computed from $\mathcal{L}_\Lambda$ in (2) easily

$$\begin{aligned} V_{tree} = {} & (h^2|\phi_N|^2 + m_1^2)|\phi_{H_1}|^2 + (h^2|\phi_N|^2 + m_2^2)|\phi_{H_2}|^2 + m_M^2|\phi_M|^2 + m^2|\phi_N|^2 \\ & + (m_T^2 + f_T^2|\phi_{H_2}|^2)(|\phi_Q|^2 + |\phi_{T_C}|^2) + f_T^2|\phi_Q|^2|\phi_{T_C}|^2 \\ & + (h\phi_N\phi_{H_1})(f_T\phi_Q^\dagger\phi_{T_C}^\dagger) + (h\phi_N^\dagger\phi_{H_1}^\dagger)(f_T\phi_Q\phi_{T_C}) \\ & + mB\phi_M\phi_N + mB\phi_M^\dagger\phi_N^\dagger + \frac{m_{12}^2}{\sqrt{G}}\phi_M + \frac{m_{12}^2}{\sqrt{G}}\phi_M^\dagger \\ & - F_N(m\phi_M + h\phi_{H_1}\phi_{H_2}) - F_N^\dagger(m\phi_M^\dagger + h\phi_{H_1}^\dagger\phi_{H_2}^\dagger) \end{aligned} \tag{15}$$

According to [25], we then compute the field-dependent squared-mass matrices, $\hat{M}_B^2$ and $\hat{M}_F^2$, and the results are summarized as follows:
$\hat{M}_F^2 =$

$$\begin{pmatrix} h^2|\phi_N|^2 & 0 & (h\phi_N^\dagger)(f_T\phi_{T_C}) & (h\phi_N^\dagger)(f_T\phi_Q) \\ 0 & \begin{matrix} h^2|\phi_N|^2 \\ +f_T^2(|\phi_Q|^2+|\phi_{T_C}|^2) \end{matrix} & f_T^2\phi_{H_2}\phi_Q^\dagger & f_T^2\phi_{H_2}\phi_{T_C}^\dagger \\ (h\phi_N)(f_T\phi_{T_C}^\dagger) & f_T^2\phi_{H_2}^\dagger\phi_Q & f_T^2(|\phi_{H_2}|^2+|\phi_{T_C}|^2) & f_T^2\phi_Q\phi_{T_C}^\dagger \\ (h\phi_N)(f_T\phi_Q^\dagger) & f_T^2\phi_{H_2}^\dagger\phi_{T_C} & f_T^2\phi_Q^\dagger\phi_{T_C} & f_T^2(|\phi_{H_2}|^2+|\phi_Q|^2) \end{pmatrix} \tag{16}$$

$$\hat{M}_B^2 = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{B}^\dagger & \hat{C} \end{pmatrix} \tag{17}$$

$$\hat{A} =$$

$$\begin{pmatrix} h^2|\phi_N|^2+m_1^2 & -hF_N^\dagger & 0 & 0 \\ -hF_N & \begin{matrix} h^2|\phi_N|^2+m_2^2 \\ +f_T^2(|\phi_Q|^2+|\phi_{T_C}|^2) \end{matrix} & 0 & 0 \\ 0 & 0 & \begin{matrix} h^2|\phi_N|^2+m_2^2 \\ +f_T^2(|\phi_Q|^2+|\phi_{T_C}|^2) \end{matrix} & -hF_N^\dagger \\ 0 & 0 & -hF_N & h^2|\phi_N|^2+m_1^2 \end{pmatrix} \tag{18}$$

$$\hat{B} =$$

$$\begin{pmatrix} (h\phi_N^\dagger)(f_T\phi_{T_C}) & 0 & (h\phi_N^\dagger)(f_T\phi_Q) & 0 \\ f_T^2\phi_{H_2}^\dagger\phi_Q^\dagger & f_T^2\phi_{H_2}^\dagger\phi_{T_C} & f_T^2\phi_{H_2}^\dagger\phi_{T_C}^\dagger & f_T^2\phi_{H_2}^\dagger\phi_Q \\ f_T^2\phi_{H_2}\phi_Q^\dagger & f_T^2\phi_{H_2}\phi_{T_C} & f_T^2\phi_{H_2}\phi_{T_C}^\dagger & f_T^2\phi_{H_2}\phi_Q \\ 0 & (h\phi_N)(f_T\phi_Q^\dagger) & 0 & (h\phi_N)(f_T\phi_{T_C}^\dagger) \end{pmatrix} \tag{19}$$

$\hat{C} =$

$$\begin{pmatrix} \begin{array}{l} f_T^2(|\phi_{H_2}|^2 + |\phi_{T_C}|^2) \\ +m_T^2 \end{array} & \begin{array}{l} (h\phi_N)(f_T\phi_{H_1}) \\ +f_T^2\phi_Q\phi_{T_C} \end{array} & f_T^2\phi_Q\phi_{T_C}^\dagger & 0 \\ \begin{array}{l} (h\phi_N^\dagger)(f_T\phi_{H_1}^\dagger) \\ +f_T^2\phi_Q^\dagger\phi_{T_C}^\dagger \end{array} & \begin{array}{l} f_T^2(|\phi_{H_2}|^2 + |\phi_Q|^2) \\ +m_T^2 \end{array} & 0 & f_T^2\phi_Q\phi_{T_C}^\dagger \\ f_T^2\phi_Q^\dagger\phi_{T_C} & 0 & \begin{array}{l} f_T^2(|\phi_{H_2}|^2 + |\phi_Q|^2) \\ +m_T^2 \end{array} & \begin{array}{l} (h\phi_N)(f_T\phi_{H_1}) \\ +f_T^2\phi_Q\phi_{T_C} \end{array} \\ 0 & f_T^2\phi_Q^\dagger\phi_{T_C} & \begin{array}{l} (h\phi_N^\dagger)(f_T\phi_{H_1}^\dagger) \\ +f_T^2\phi_Q^\dagger\phi_{T_C}^\dagger \end{array} & \begin{array}{l} f_T^2(|\phi_{H_2}|^2 + |\phi_{T_C}|^2) \\ +m_T^2 \end{array} \end{pmatrix} \tag{20}$$

(14) can be integrated to give the following:

$$V_{1-loop} \quad = \quad \frac{1}{32\pi^2}\{ \, \Lambda^2 STr(\hat{M}^2) + \frac{1}{2}STr(\hat{M}^4 \ln(\frac{\hat{M}^2}{\Lambda^2})) - \frac{1}{4}STr(\hat{M}^4) + O(\frac{\hat{M}^6}{\Lambda^2}) \, \} \tag{21}$$

In general, $\hat{M}^2$ is of order F_{SUSY}^2, and therefore it's reasonable to neglect the last term in (21) because $F_{SUSY} \ll \Lambda$ in reality. The first term in (21) is also neglected because $STr(\hat{M}^2)$ depends only on the soft SUSY-breaking parameters, which is just a constant contribution to V_{eff}. Denote the eigenvalues of $\hat{M}_B^2$ and $\hat{M}_F^2$ as ω_{Bi}^2 and ω_{Fi}^2 respectively, and V_{1-loop} can be written as follows:

$$\begin{aligned} V_{1-loop} \quad = \quad & \frac{1}{32\pi^2}\sum_{Bi=1}^{8}\{ \, \frac{1}{2}\omega_{Bi}^4 \ln(\frac{\omega_{Bi}^2}{\Lambda^2}) - \frac{1}{4}\omega_{Bi}^4 \, \} \\ & -\frac{1}{16\pi^2}\sum_{Fi=1}^{4}\{ \, \frac{1}{2}\omega_{Fi}^4 \ln(\frac{\omega_{Fi}^2}{\Lambda^2}) - \frac{1}{4}\omega_{Fi}^4 \, \} \end{aligned} \tag{22}$$

In general, $\hat{M}_B^2$ and $\hat{M}_F^2$ are non-trivial large matrices, and therefore numerical methods are needed in order to compute ω_{Bi}^2 and ω_{Fi}^2. The above results will be useful later. In fact, our computations of V_{eff} will be verified in Section 4 by comparing a special case of ours with that of [12]. The extension of the computations (15)−(22) to the MSMHHC of SU(2)×U(1) has been worked out, too. However, this extension is straightforward, and therefore it is not given here.

4 The Vacuum of the MSMHHC

Due to phenomenological considerations, we are interested in the vacuum configuration of the MSMHHC where ϕ_{H_1}, ϕ_{H_2}, ϕ_M and ϕ_N develop non-trivial VEV's, but the other scalar fields do not. That is, the SU(2)×U(1) → U(1) electroweak symmetry breaking accompanies the Higgs-Higgs condensation for the MSMHHC of SU(2)×U(1) symmetry. For the MSMHHC of U(1) symmetry, this vacuum configuration means that the spontaneous breaking of U(1) accompanies the Higgs-Higgs condensation. This MSMHHC vacuum is obtained by finding the extremum of V_{eff}. Thanks to $< \phi_Q >=< \phi_{T_C} >= 0$, the extremization of V_{eff} can be performed exactly, and it leads to a set of five vacuum constraints. The imaginary parts of these vacuum constraints are used to fix the relative phases among different VEV's. The above computations have been performed for the MSMHHC of U(1) symmetry and the results are as follows. In this paper, we always take m, h, B and f_T to be positive without loss of generality.

$$\begin{aligned} < \phi_{H_1} > &= v_1,\ < \phi_{H_2} >= v_2 e^{i\varphi},\ < \phi_M >= -v_M e^{i\varphi}, \\ < \phi_N > &= v_N e^{-i\varphi},\ < F_N >= v_{F_N} e^{-i\varphi}, \\ < \phi_Q > &= < \phi_{T_C} >= 0 \end{aligned} \tag{23}$$

where v_1, v_2, v_M, v_N and v_{F_N} are positive. From now on, (23) will be called the MSMHHC vacuum, and we always choose the special case: $\varphi = 0$. When $\varphi = 0$, notice that $< \phi_M >$ and $< \phi_N >$ are exactly out of phase. With this MSMHHC vacuum, there are still five real vacuum constraints, (24)−(28), coming from the variation of V_{eff} (the effective potential of the MSMHHC of U(1), (15)−(22)) with respect to v_1, v_2, v_M, v_N and v_{F_N} respectively.

$$h v_{F_N} v_2 = (h^2 v_N^2 + m_1^2) v_1 - \frac{f_T}{32\pi^2}(h v_N)\{\, \omega_3^2 \ln(\frac{\Lambda^2}{\omega_3^2}) - \omega_4^2 \ln(\frac{\Lambda^2}{\omega_4^2})\,\} \tag{24}$$

$$\begin{aligned} h v_{F_N} v_1 = (h^2 v_N^2 + m_2^2) v_2 - \frac{f_T^2 v_2}{16\pi^2}\{\, \omega_3^2 \ln(\frac{\Lambda^2}{\omega_3^2}) + \omega_4^2 \ln(\frac{\Lambda^2}{\omega_4^2}) \\ - 2\omega_6^2 \ln(\frac{\Lambda^2}{\omega_6^2})\,\} \end{aligned} \tag{25}$$

$$h v_{F_N} = -\sqrt{G} m_M^2 v_M + B(h v_N) + m_{12}^2 \tag{26}$$

$$
\begin{aligned}
-\ \frac{B}{\sqrt{G}} v_M \ &+\ h v_N (v_1^2 + v_2^2 + \frac{1}{G})\ -\ \frac{h v_N}{16\pi^2} \{\, \omega_1^2 \ln(\frac{\Lambda^2}{\omega_1^2})\ +\ \omega_2^2 \ln(\frac{\Lambda^2}{\omega_2^2}) \\
&-\ 2\omega_5^2 \ln(\frac{\Lambda^2}{\omega_5^2})\ +\ (\frac{f_T v_1}{2 h v_N}) \omega_3^2 \ln(\frac{\Lambda^2}{\omega_3^2})\ -\ (\frac{f_T v_1}{2 h v_N}) \omega_4^2 \ln(\frac{\Lambda^2}{\omega_4^2})\,\}\ = 0
\end{aligned}
\tag{27}
$$

$$
v_1 v_2 \ -\ \frac{1}{\sqrt{G}} v_M \ +\ \frac{1}{32\pi^2} (\frac{h v_{F_N}}{\sqrt{\frac{1}{4}(m_1^2 - m_2^2)^2 + h^2 v_{F_N}^2}}) \{\, \omega_1^2 \ln(\frac{\Lambda^2}{\omega_1^2})\ -\ \omega_2^2 \ln(\frac{\Lambda^2}{\omega_2^2})\,\}\ = 0 \tag{28}
$$

where

$$
\omega_1^2 \ = \ h^2 v_N^2 + \frac{1}{2}(m_1^2 + m_2^2) + \sqrt{\frac{1}{4}(m_1^2 - m_2^2)^2 + h^2 v_{F_N}^2} \tag{29}
$$

$$
\omega_2^2 \ = \ h^2 v_N^2 + \frac{1}{2}(m_1^2 + m_2^2) - \sqrt{\frac{1}{4}(m_1^2 - m_2^2)^2 + h^2 v_{F_N}^2} \tag{30}
$$

$$
\omega_3^2 \ = \ f_T^2 v_2^2 + m_T^2 + (h v_N)(f_T v_1) \tag{31}
$$

$$
\omega_4^2 \ = \ f_T^2 v_2^2 + m_T^2 - (h v_N)(f_T v_1) \tag{32}
$$

$$
\omega_5^2 \ = \ h^2 v_N^2 \tag{33}
$$

$$
\omega_6^2 \ = \ f_T^2 v_2^2 \tag{34}
$$

(24)$-$(34) uniquely define the study of the MSMHHC vacuum for the MSMHHC of U(1) symmetry. The eigenvalues (of multiplicity 2) of $\hat{M}_B^2$ and $\hat{M}_F^2$ evaluated at the MSMHHC vacuum are denoted here as $(\omega_1^2, \ldots, \omega_4^2)$ and $(\omega_5^2,\, \omega_6^2)$ respectively. Numerical methods are needed in order to study (24)$-$(34).

In comparison with the usual SUSY NJL model (for example, the MTCSSM [12]), the MSMHHC has two more vacuum constraints, i.e., the constraints (24) and (25), which corresponds to the fact that the constituent fields (H_1 and H_2) of the condensates also develop non-trivial VEV's (v_1 and v_2) in the formulation of the MSMHHC. In the MTCSSM, the constituent fields of the condensates are the top superfields, which do not develop non-trivial VEV's. Therefore, when $v_1 = v_2 = 0$, constraints (24) and (25) are washed away and we recover the usual results. If $f_T = 0$ and $B = m_{12} = 0$ are further assumed, the constraints (26)$-$(28) will be the same as those obtained in [12] for the MTCSSM essentially, and G will satisfy the usual SUSY gap equation [12].

However, with $v_1,\ v_2 \neq 0$, the MSMHHC is indeed more complicated than the usual SUSY NJL model. Due to the unique feature that the MSMHHC has

two more vacuum constraints, an immediate implication is that the MSMHHC should be more predictive than the usual SUSY NJL model (e.g., the MTCSSM). In fact, there is one more physical constraint for the MSMHHC: $\sqrt{v_{1r}^2 + v_{2r}^2} = \frac{250}{\sqrt{2}}$ GeV at low energy. Therefore, together with the constraints (24)−(28), the independent input parameters required by the MSMHHC are:

$$\Lambda,\ f_T \text{ and } \ (\ m_1,\ m_2,\ m_M,\ m_T,\ m_{12},\ B\ ) \tag{35}$$

Once (35) is specified, everything is determined. Notice that the predictions of the MSMHHC are essentially determined by the soft SUSY-breaking terms only. With (35) specified, the determination of the following quantities is always interesting:

$$G,\ \ v_M,\ \ hv_N,\ \ hv_{F_N},\ \ \tan\beta = \frac{v_2}{v_1} \tag{36}$$

Unlike the MTCSSM, the four-Fermi coupling G is determined from (35) and is not a free parameter in MSMHHC. This suggests that, within the framework of the MSMHHC, the origin of the four-field interaction, $G(H_1\epsilon H_2)^\dagger(H_1\epsilon H_2)$, should be closely related to the supersymmetry breaking. A systematic numerical study of the MSMHHC vacuum throughout the parameter space will be given in Sections 6−9.

5 The Mass Spectrum and the Lightest Higgs Boson

To determine the full spectrum of the scalar particles, we need to compute the full squared-mass matrix from V_{eff}, that is, the second derivatives of V_{eff} evaluated at the MSMHHC vacuum. Due to the explicit dependence of V_{eff} on the auxiliary field F_N, the computation of the second derivative is a little complicated:

$$
\begin{aligned}
\frac{d^2V_{eff}}{dAdB} &= \frac{\partial^2 V_{eff}}{\partial A\partial B} + \frac{\partial^2 V_{eff}}{\partial A\partial F_N}(\frac{\partial F_N}{\partial B}) + \frac{\partial^2 V_{eff}}{\partial A\partial F_N^\dagger}(\frac{\partial F_N^\dagger}{\partial B}) \\
&+ \frac{\partial^2 V_{eff}}{\partial B\partial F_N}(\frac{\partial F_N}{\partial A}) + \frac{\partial^2 V_{eff}}{\partial B\partial F_N^\dagger}(\frac{\partial F_N^\dagger}{\partial A}) \\
&+ \frac{\partial^2 V_{eff}}{\partial F_N^2}(\frac{\partial F_N}{\partial A})(\frac{\partial F_N}{\partial B}) + \frac{\partial^2 V_{eff}}{\partial F_N^{\dagger^2}}(\frac{\partial F_N^\dagger}{\partial A})(\frac{\partial F_N^\dagger}{\partial B}) \\
&+ \frac{\partial^2 V_{eff}}{\partial F_N\partial F_N^\dagger}[\,(\frac{\partial F_N}{\partial A})(\frac{\partial F_N^\dagger}{\partial B}) + (\frac{\partial F_N^\dagger}{\partial A})(\frac{\partial F_N}{\partial B})\,] \qquad (37)
\end{aligned}
$$

where A and B represent the scalar fields ϕ_{H_1}, ϕ_{H_2}, ϕ_M, ϕ_N, ϕ_Q, ϕ_{T_C}, or their complex conjugates. The constraint $\frac{\partial V_{eff}}{\partial F_N} = 0$ has been used in deriving (37). Terms like $\frac{\partial F_N}{\partial A}$ can be obtained by differentiating $\frac{\partial V_{eff}}{\partial F_N} = 0$. However, (37) is useful only if eigenvalues of the large matrices involved in V_{eff} (22) can be solved exactly. In general, V_{eff} and its second derivatives have to be computed by numerical methods. These numerical computations are time-consuming because, given any scalar field configuration, the evaluation of V_{eff} requires solving a 2-dim extremization problem numerically due to the constraint $\frac{\partial V_{eff}}{\partial F_N} = 0$. More details about the computation of the mass spectrum will be given in Section 7. As expected, the numerically computed mass spectrum for the MSMHHC of U(1) contains exactly one massless particle due to the broken U(1) symmetry. We have also computed the mass spectrum for the MSMHHC of SU(2)×U(1), and there are three massless particles, i.e., three Goldstone bosons which will be absorbed by gauge bosons if SU(2)×U(1) is gauged. The systematic numerical study of the mass spectrum for the MSMHHC of U(1) symmetry throughout the parameter space will be given in Sections 7−9.

Because the mass of the lightest Higgs boson (m_{LH}) is one of our main concerns, it's definitely important to ask how m_{LH} depends on the top Yukawa interaction. The best way to answer this question is to turn off the top Yukawa interaction ($f_T = 0$), and therefore the MSMHHC of U(1) is reduced to a pure Higgs sector. To reveal a unique feature of the MSMHHC of U(1) with $f_T = 0$, we write down (24), (25) and (30) with f_T set to zero:

$$
h v_{F_N} v_2 = (h^2 v_N^2 + m_1^2) v_1 \qquad (38)
$$

$$
h v_{F_N} v_1 = (h^2 v_N^2 + m_2^2) v_2 \qquad (39)
$$

$$\omega_2^2 \;=\; h^2 v_N^2 + \frac{1}{2}(m_1^2 + m_2^2) - \sqrt{\frac{1}{4}(m_1^2 - m_2^2)^2 + h^2 v_{F_N}^2} \qquad (40)$$

(38) and (39) are the vacuum constraints corresponding to the variations of V_{eff} with respect to v_1 and v_2, which do not exist in the usual SUSY NJL model. Notice that, with $f_T = 0$, (38) and (39) are actually the tree-level vacuum constraints, and remain unaffected by one-loop corrections. ω_2^2 is an eigenvalue of the tree-level squared-mass matrix $\hat{M}_B^2$. Remember that we have been working with the MSMHHC of U(1), and therefore ω_2^2 in (40) has multiplicity 2. Notice that the validity of (38)$-$(40) is not limited the MSMHHC of U(1). In fact, it is straightforward to verify that (38)$-$(40) also hold for the MSMHHC of SU(2)$\times$U(1), and ω_2^2 has multiplicity 4 in the case of SU(2)$\times$U(1). Combining (38) and (39) with (40), one has $\omega_2^2 = 0$. It means that, at tree level, there are two massless particles for the MSMHHC of U(1) (or, four massless particles for the MSMHHC of SU(2)$\times$U(1)). In either case, there is one more massless particle than what is usually expected. However, the above tree-level results are not conclusive, and we must resort to one-loop contributions, $V_{eff} = V_{tree} + V_{1-loop}$.

With $f_T = 0$, V_{eff} can be computed exactly, and (37) is useful in computing the second derivatives of V_{eff} at the vacuum. Because these computations are trivial and lengthy, their details are not presented here. The full spectrum is then obtained by solving the eigenvalues of the full squared-mass matrix, which is composed of the second derivatives of V_{eff} evaluated at the vacuum. Our computations for the MSMHHC of U(1) indicates that certain one-loop contributions cancel during the computation, and there are still two massless particles. We have also computed the case of SU(2)$\times$U(1), and there are four massless particles. That is, besides the Goldstone bosons, there is always one more massless particle for the MSMHHC of either U(1) or SU(2)$\times$U(1), where this additional massless particle is just the lightest Higgs. Therefore, the unique feature of the MSMHHC with $f_T = 0$ is: the lightest Higgs remains massless, at least to one-loop order. Although we are not able to give a formal proof of the above result in a few sentences, this unique feature is indeed well established by the reliable computations based on (37) and (15)$-$(22).

An obvious way to make the lightest Higgs massive is to violate the vacuum constraints (38) and (39). If they are violated, then $\omega_2^2 = 0$ will no longer hold, and we shall have exactly one massless particle for the MSMHHC of U(1) (or,

three massless particles for the MSMHHC of SU(2)×U(1)). Furthermore, the lightest Higgs will receive a mass proportional to the amount of violation if it is small. According to (24) and (25), turning on the top Yukawa interaction does violate (38) and (39). Therefore, the violation of $\omega_2^2 = 0$ due to f_T is roughly proportional to $f_T^2(h^2 v_N^2 \sigma_1 + 2m_T^2 \sigma_2)$, where

$$\begin{aligned} \sigma_1 &= \frac{h^2 v_N^2 + m_2^2}{h^2 v_N^2 + \frac{1}{2}(m_1^2 + m_2^2)} \\ \sigma_2 &= \frac{h^2 v_N^2 + m_1^2}{h^2 v_N^2 + \frac{1}{2}(m_1^2 + m_2^2)} \end{aligned} \tag{41}$$

So, we have the following rough qualitative estimate for m_{LH}:

$$m_{LH}^2 \propto f_T^2(h^2 v_N^2 \sigma_1 + 2m_T^2 \sigma_2) \tag{42}$$

This qualitative estimate is good when the amount of violation (i.e., m_{LH}) is not very large, and the above argument indicates that (42) is shared by both the MSMHHC of U(1) and the MSMHHC of SU(2)×U(1). (42) will be shown to be compatible with the results obtained in Sections 8 and 9. (42) shows that m_{LH} is proportional to the mass of the top quark, which reveals an important feature of the MSMHHC: heavy top quark always implies comparatively large m_{LH}. This is really a nice feature from the viewpoint of the second issue. (42) also indicates that m_{LH} is proportional to F_{SUSY} (the strength of soft SUSY breaking). As we shall see in Section 8, m_{LH} is indeed proportional to F_{SUSY}, and saturates when F_{SUSY} becomes very large.

6 The Parameter Space and Low-Energy Quantities

In Sections 6−9, we always work with the MSMHHC of U(1). However, there is no loss of generality because the qualitative features obtained in Sections 6−9 are shared by both the MSMHHC of U(1) and the MSMHHC of SU(2)×U(1). Therefore, the MSMHHC of U(1) symmetry won't be emphasized unless it is necessary. For convenience, we always specify the parameter space at Λ according to (35). That is, Λ, f_T and (m_1, m_2, m_M, m_T, m_{12}, B) define the full parameter space of the MSMHHC at the scale Λ, where all the parameters are taken to be positive without loss of generality. At any point of this parameter

space, we first compute the relevant physical quantities at Λ, and then renormalize all the parameters and physical quantities down to low energy properly according to (9)−(13). The low-energy ones are distinguished from the ones at Λ by a subscript r, and therefore care should be taken. A systematic study of the full parameter space is not feasible unless reasonable physical constraints are imposed in order to reduce the number of free parameters. Next, we shall discuss the physical constraints assumed here one by one.

The first reasonable constraint is to require that all the low-energy soft SUSY-breaking scalar squared masses be non-negative except for the induced scalar squared mass $m^2_{N_r}$. When applied to (10), it leads to two non-trivial constraints: $m^2_{2r} \geq 0$ and $m^2_{T_r} \geq 0$, where $m^2_{T_r} = m^2_{Q_r} = m^2_{T_{Cr}}$ and $\Sigma_T = \Sigma_Q = \Sigma_{T_C} = \Sigma_{H_2}$ due to $N_W = 1$. These two constraints can be rewritten in terms of Σ_T:

$$\Sigma_T \leq \frac{m_2^2}{2m_T^2} \quad \text{and} \quad \Sigma_T \leq \frac{m_T^2}{m_2^2 + m_T^2},$$
$$\Sigma_T = \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}) \tag{43}$$

where μ_E is chosen to be the low-energy scale, i.e., $\mu_E \ll \Lambda$. In the case of either $m_2^2 \gg m_T^2$ or $m_2^2 \ll m_T^2$, (43) requires $\Sigma_T = \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}) \ll 1$. $\frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}) \ll 1$ is not favored by phenomenology because it may lead to small f_T, and small f_T contradicts with the present observation of heavy top quark. The absolute upper bound on Σ_T can be derived from (43):

$$\Sigma_T = \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}) \;\leq\; \frac{1}{2}, \quad \text{and}$$
$$\Sigma_T = \frac{f_T^2}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2}) \;=\; \frac{1}{2} \quad \text{when} \quad m_2^2 = m_T^2. \tag{44}$$

Due to the fact that top quark is heavy, the first constraint requires us to choose $m_2^2 \approx m_T^2$ naturally. In practice, it is found that $\frac{1}{2} < \frac{m_2}{m_T} < 2$ is required when the physical top-quark mass $m_{top} = 180$ GeV. Therefore, $m_2^2 = m_T^2$ is chosen in the study of the parameter space.

(44) leads to another important result when it is combined with (13) ($\Sigma_{H_2} = \Sigma_T$):

$$h_r \;=\; \frac{1}{\sqrt{1+\Sigma_T}} \cdot \frac{1}{\sqrt{\frac{1}{16\pi^2} \ln(\frac{\Lambda^2}{\mu_E^2})}} \;\geq\; \frac{2}{\sqrt{3}} f_T \tag{45}$$

(45) implies that heavy top quark (i.e., large f_T) always leads to strongly-interacting low-energy Higgs sector (i.e., large h_r), a unique feature of MSMHHC. Since a strongly-interacting low-energy Higgs sector always leads to large m_{LH}, (45) actually re-establishes the conclusion obtained in Section 5.

The second constraint is to fix the physical top-quark mass m_{top} as 164 GeV $\sim$ 180 GeV, and therefore f_T is fixed in the study of the parameter space. Generally, it is found that $1 < f_T < 2$. Therefore, (44) can be interpreted in a different way by rewriting it as follows:

$$\Lambda \ \leq \ \mu_E \cdot e^{\frac{4\pi^2}{f_T^2}} \tag{46}$$

(46) leads to an upper bound on Λ and, due to the second constraint ($1 < f_T < 2$), it implies that the MSMHHC should be an effective intermediate-scale model, i.e., weak scale $\ll \Lambda \ll$ Planck scale. Finally, we shall fix m_M^2 by $m_M^2 = m_1^2 + m_2^2$. According to (4), M is the composite of H_1 and H_2. Therefore, it is reasonable to assume that the SUSY breaking felt by M is the sum of those felt by H_1 and H_2 [10, 11].

However, there is a fine-tuning problem associated with the soft SUSY-breaking parameters. One way to express this fine-tuning problem is to fix all the parameters except for B. (24)$-$(28) are then solved numerically with respect to different choices of B, and the results indicate that B must satisfy the following inequality:

$$B_{lower} \ \leq \ B \ < \ B_{upper} \tag{47}$$

where B_{upper} and B_{lower} depend on all the parameters except for B. In Fig.2, (47) is plotted with respect to different choices of F_{SUSY}, where the other parameters are chosen as: $\Lambda = 2.5 \times 10^4$ TeV, $f_T = 1.6$, $m_1 = 2\,F_{SUSY}$, $m_2 = m_T = F_{SUSY}$, $m_M = \sqrt{5}\,F_{SUSY}$, $m_{12} = \frac{1}{2}\,F_{SUSY}$. $B \geq B_{lower}$ is a gap equation, and therefore this lower bound is natural. That is, in studying (24)$-$(28), $B < B_{lower}$ results in no solution. (24)$-$(28) will result in exactly one solution for the vacuum if $B = B_{lower}$, and two solutions if $B > B_{lower}$. However, by checking the mass spectrum, the vacuum will develop instability if $B > B_{upper}$. In general, $\frac{B_{upper} - B_{lower}}{B} \approx \frac{5}{100}$. Therefore, the upper bound B_{upper} is unnatural and B needs fine tuning. This fine-tuning problem arises mainly due to the fact

that the MSMHHC has two more vacuum constraints (24) and (25). Since this fine tuning is not severe, it suggests that a better choice of non-renormalizable interactions may be able to relax this fine tuning by modifying the vacuum constraints (24)−(28) properly. The freedom of choosing non-renormalizable interactions without violating symmetries has been an objection to the model of top condensation because this arbitrariness undermines its predictive power [21, 26]. The same objection applies to MSMHHC. However, by the requirement of no fine tuning, the possible choices of non-renormalizable interactions can be highly reduced, and therefore this objection may be partially resolved. From this viewpoint, this fine-tuning problem is a useful guide to the future model-building of the MSMHHC rather than an obstacle. For the present, we simply choose $B = B_{lower}$ in the study of the parameter space. According to this choice, m_{LH} computed in Sections 7−9 is actually the upper bound.

According to the above constraints and choices, the parameter space under study contains only four free parameters:

$$\Lambda \text{ and } (\ m_1,\ m_2,\ m_{12}\ ) \tag{48}$$

where the other parameters are determined by:

$$m_T = m_2,\ \ m_M = \sqrt{m_1^2 + m_2^2},\ \ B = B_{lower} \tag{49}$$

and f_T is determined by the requirement that the physical top-quark mass m_{top} be 164 GeV $\sim$ 180 GeV. Based on (48) and (49), a systematic study of the parameter space will be given in Sections 7−9.

We have described in Sections 3−6 how to compute the physics of the MSMHHC using the effective potential. However, the approach using RGE's is adopted in many studies of the top-condensate models [10, 11]. Therefore, it is worth mentioning the RGE approach to the MSMHHC and comparing it with the present approach. It has been shown that the low-energy effective theory of the MSMHHC is the (M+2)SSM, whose details are given in (8)−(13). The spirit of the RGE approach is to work with the low-energy theory (M+2)SSM instead of the MSMHHC, and to translate the high-energy features of MSMHHC into the high-energy boundary condition of the (M+2)SSM RGE's. That is, all the RGE approach needs is just the (M+2)SSM and its boundary condition at the

cut-off Λ. According to (12) and (13), it is obvious that the correct boundary condition (the compositeness condition) for the (M+2)SSM is:

$$h_r = \infty, \;\; m_r = \infty \;\text{ and }\; \frac{h_r}{m_r} = \text{finite} \;\; \text{at} \;\; \mu_E = \Lambda. \tag{50}$$

As discussed in Section 2, this compositeness condition can be viewed as the triviality problem of the (M+2)SSM, which indicates the existence of composite fields in the (M+2)SSM. Besides, the h_r computed from the compositeness condition is just the so-called "triviality bound" [8, 20]. Therefore, through the RGE approach, the study of triviality bounds merges in the study of condensate models nicely. As for technical aspects, it's easier to implement the RGE approach than the effective potential approach, especially when the full content of the standard model is included. These viewpoints do make the (M+2)SSM an interesting low-energy model. We plan to publish the study of the RGE approach to the MSMHHC (including gauge fields) and the study of triviality bounds in the (M+2)SSM in a separate paper.

However, it's necessary to explain why the RGE approach is not adopted in the present paper in order to emphasize the physical significance of the effective potential approach. We may start with the question: "Can the RGE approach faithfully embrace all the physical features of the high-energy condensate model in a natural way?" Since there is no one-to-one correspondence between the low-energy effective models and the high-energy models, the answer is no because the RGE approach is based only on the low-energy effective model with the compositeness condition, and the compositeness condition alone is not enough to embrace all the physical features of the high-energy condensate model. For example, the SUSY gap equation, which is essential in our solution to the μ problem, does not come out of the RGE approach naturally. On the other hand, the effective potential approach is the natural and faithful way to express the high-energy physics. Since the Higgs-Higgs condensation is a new idea, we choose the effective potential approach in the present paper in order to give a complete presentation of the physics.

7 The Dependence of Low-Energy Quantities on the Cut-off Λ

The general computational procedure has been described in detail, and its basic steps are summarized as follows. Given a point of the parameter space according to (48) and (49), the first step is to solve (24)−(28) for the MSMHHC vacuum numerically, and therefore all the non-trivial VEV's, G and B_{lower} are determined. Next, according to Sections 3 and 5, the full mass spectrum is computed. Finally, according to (9)−(13), all the physical quantities and parameters are renormalized down to the low-energy scale $\mu_E = v_{N_r}$, where v_{N_r} is of order the physical mass of the lightest Higgs m_{LH}. We always require m_{top}=164 or 180 GeV. (m_{top}: the physical mass of top quark), and therefore the choice of f_T should be consistent with this m_{top}.

The dependence on Λ is illustrated with a typical example, where its parameters are chosen as: $F_{SUSY} = 800$ GeV, $m_1 = 2\,F_{SUSY}$, $m_2 = F_{SUSY}$, $m_{12} = \frac{1}{2}\,F_{SUSY}$, m_{top}=164 or 180 GeV, and Λ is left free. The physical mass of the lightest Higgs boson m_{LH} versus the cut-off Λ is plotted in Fig.3. With $10^4\,\text{GeV} < \Lambda < 10^8\,\text{GeV}$, the mass of the lightest Higgs can be as large as $150\,\text{GeV} < m_{LH} < 400\,\text{GeV}$ in Fig.3. Compared to the MSSM or the NMSSM, the MSMHHC is indeed able to predict much larger m_{LH}. Therefore, the MSMHHC widens the Higgs search, and should be taken more seriously if the Higgs signal is still absent in the near future. In general, m_{LH} decreases as Λ increases. However, it is clear from Fig.3 that Λ is bounded from above. This upper bound on Λ (Λ_{upper}) arises due to the inequality (46), i.e., there will be negative soft SUSY-breaking squared masses at the low-energy scale $\mu_E = v_{N_r}$ if $\Lambda > \Lambda_{upper}$. It explicitly confirms the conclusion obtained in Section 6 that the MSMHHC should be an effective intermediate-scale model. In addition, this upper bound on Λ also implies a lower bound on m_{LH}, which is consistent with (45). That is, as f_T increases, Λ_{upper} decreases according to (46), and therefore the lower bound on m_{LH} increases according to Fig.3.

As shown in Fig.3, the prediction of m_{LH} is insensitive to the choice of m_{top} within 164 GeV $\leq m_{top} \leq$ 180 GeV unless Λ is close to Λ_{upper}. Therefore, in Sections 8 and 9, only the case of $m_{top} = 180$ GeV will be studied. Notice that (42) is not applicable to Fig.3 because (42) is valid only when m_{LH} is compara-

tively small. The predictions of Fig.3 do lie outside the linear region described by (42). It will be shown in Fig.6 of Section 8 that the predictions of Fig.3 lie in the saturated region due to the comparatively large choice $F_{SUSY} = 800$ GeV. Due to saturation, m_{LH} depends on f_T mainly through the renormalization effects, not (42). Therefore, it explains why m_{LH} predicted by $m_{top} = 180$ GeV is slightly smaller than that predicted by $m_{top} = 164$ GeV.

$\tan\beta_r = \frac{v_{2r}}{v_{1r}}$ versus Λ is plotted in Fig.4. Due to the choice $\frac{m_1}{m_2} = 2$, $\tan\beta_r$ lies between $1.5 \sim 3.5$ and slightly increases as Λ increases. As a rough approximation, $\tan\beta_r$ is obtained from (24) and (25) by turning f_T off:

$$\tan\beta_r = \frac{v_{2r}}{v_{1r}} \approx \sqrt{\frac{m_1^2 + h^2 v_N^2}{m_2^2 + h^2 v_N^2}} \tag{51}$$

Therefore, $\tan\beta_r$ is essentially determined by the ratio $\frac{m_1}{m_2}$. Different choices of $\frac{m_1}{m_2}$ will be made in Section 9. In Fig.5, the plot of $\mu_r = h_r < \phi_{N_r} >$ versus Λ is given. μ_r decreases as Λ increases and, as expected, μ_r is always of order $F_{SUSY} = 800$ GeV, the soft SUSY-breaking strength.

8 The Dependence of Low-Energy Quantities on F_{SUSY}

In this section, we would like to study the dependence on the soft SUSY-breaking strength F_{SUSY} by fixing the soft SUSY-breaking pattern (i.e., keeping the ratio between any two soft SUSY-breaking parameters fixed). The dependence on F_{SUSY} is illustrated with a typical example, where its parameters are chosen as: $\Lambda = 500$ TeV, $m_1 = 2\,F_{SUSY}$, $m_2 = F_{SUSY}$, $m_{12} = \frac{1}{2}\,F_{SUSY}$, m_{top}=180 GeV, and F_{SUSY} is left free. Following the computational procedure described in Section 7, m_{LH} versus F_{SUSY} is plotted in Fig.6. When F_{SUSY} is small (e.g., $F_{SUSY} < 500$ GeV for Fig.6), m_{LH} is almost proportional to F_{SUSY}, which has been predicted by (42). When F_{SUSY} is large (e.g., $F_{SUSY} > 500$ GeV for Fig.6), (42) is no longer valid and m_{LH} actually saturates. Therefore, m_{LH} depends on F_{SUSY} in a non-trivial way and, due to the saturation of m_{LH}, the prediction of m_{LH} by the MSMHHC is stable under the variation of F_{SUSY} when F_{SUSY} is large. On the other hand, m_{LH} is sensitive to F_{SUSY} when F_{SUSY} is small and the measurement of m_{LH} can provide us with more knowledge of F_{SUSY} .

As argued in Section 2, μ_r is induced only through supersymmetry breaking effects, and therefore the μ problem is solved naturally. This argument is realized explicitly by plotting $\mu_r = h_r < \phi_{N_r} >$ versus F_{SUSY} in Fig.7. Fig.7 shows that μ_r is simply proportional to F_{SUSY} and of order F_{SUSY}, which constitutes the concrete answer to the naturalness problem of μ.

9 The Dependence of Low-Energy Quantities on the Soft SUSY-Breaking Pattern

In this section, the dependence on the soft SUSY-breaking pattern is studied. That is, Λ and F_{SUSY} are fixed, and the variation of the soft SUSY-breaking pattern means changing the ratio between any two soft SUSY-breaking parameters. First, we are interested in the dependence of the low-energy quantities on the ratio $\frac{m_1}{m_2}$ with $\sqrt{m_1^2+m_2^2}$ fixed. According to the parameter space (48) and (49), a typical example is chosen as: $\Lambda = 500$ TeV, $\sqrt{m_1^2+m_2^2} = 1$ TeV, $m_{12} = 250$ GeV, m_{top}=180 GeV, and $\frac{m_1}{m_2}$ is left free. The reason why we choose this pattern is that $\tan\beta_r$ is essentially determined by $\frac{m_1}{m_2}$ and we would like to know how physical quantities, such as m_{LH}, depend on $\tan\beta_r$. Remember that the requirement that low-energy soft SUSY-breaking scalar squared masses be non-negative leads to $m_T \approx m_2$, and therefore $m_T = m_2$ is assumed in (49). Hence, in the above example, m_T actually changes according to $m_T = m_2$ when $\frac{m_1}{m_2}$ changes. This understanding of how the soft SUSY-breaking pattern is varied in the above example is crucial to the interpretation of its results, Fig.8 and Fig.9. The above example is computed according to the procedure outlined in Section 7, and $\tan\beta_r$ versus $\frac{m_1}{m_2}$ is plotted in Fig.8. It's obvious that $\tan\beta_r \approx \frac{m_1}{m_2}$ is a good approximation to Fig.8, especially when $\frac{m_1}{m_2}$ is large. However, notice that the computations associated with $\tan\beta_r \geq O(10)$ are unsatisfactory from the viewpoint of phenomenology because all the Yukawa couplings are ignored except for the top Yukawa coupling in this paper. That is, it no longer makes sense to ignore the bottom Yukawa coupling when $\tan\beta_r \geq O(10)$. Therefore, only those results with $\tan\beta_r < O(10)$ in Fig.8 and Fig.9 are meaningful to phenomenology. Those results with $\tan\beta_r \geq O(10)$ needs to be re-computed by including the bottom Yukawa coupling. Keeping the above discussion in mind, we still consider the case $\tan\beta_r \gg 1$ in the following in order to verify the basic

features of Fig.8 and Fig.9. That is, the verification of their basic features is meant to be a consistency check only. The approximation $\tan\beta_r \approx \frac{m_1}{m_2}$ is consistent with (51), and we will show it when $\frac{m_1}{m_2} \gg 1$. Based on $\frac{m_1}{m_2} \gg 1$ and $\sqrt{m_1^2 + m_2^2}$ =constant, the vacuum constraint (24) implies $hv_{F_N} \propto \frac{1}{\tan\beta_r}$. With $hv_{F_N} \propto \frac{1}{\tan\beta_r}$, the vacuum constraint (25) implies $hv_N \propto \frac{1}{\tan\beta_r}$. When $\frac{m_1}{m_2} \gg 1$, $\tan\beta_r \gg 1$ according to Fig.8, and therefore $m_1 \gg hv_N$. With $m_1 \gg hv_N$, (51) implies that $\tan\beta_r$ is of order $\frac{m_1}{m_2}$, which is indeed consistent with Fig.8.

The above argument also leads us to the qualitative dependence of m_{LH} on $\tan\beta_r$. According to the above argument, $hv_N \propto \frac{1}{\tan\beta_r}$ and $m_T = m_2 \propto \frac{1}{\tan\beta_r}$ when $\frac{m_1}{m_2} \gg 1$. Referring to (41) and (42), one therefore has $\sigma_1 \propto \frac{1}{\tan^2\beta_r}$ and $\sigma_2 \approx 2$ when $\frac{m_1}{m_2} \gg 1$. Using (42) as a qualitative approximation to m_{LH}, one obtains the following result which shows the $\tan\beta_r$-dependence only:

$$m_{LH} \propto \sqrt{\frac{U_2}{\tan^2\beta_r} + \frac{U_1}{\tan^4\beta_r}} \tag{52}$$

where U_1 and U_2 are inessential coefficients of order unity. When $\frac{m_1}{m_2} \gg 1$, $\tan\beta_r \gg 1$ and $m_{LH} \propto \frac{1}{\tan\beta_r}$ from (52). To confirm (52), m_{LH} versus $\tan\beta_r$ is plotted in Fig.9 by changing $\frac{m_1}{m_2}$. For $1 < \tan\beta_r < 11$, 50 GeV $\leq m_{LH} \leq$ 250 GeV. It's clear from Fig.9 that m_{LH} does decrease as $\tan\beta_r$ increases, which is consistent with (52). The way m_{LH} depends on $\tan\beta_r$ has an important implication: Due to the present experimental lower bound on m_{LH} [27], large values of $\tan\beta_r$ are not favored, i.e., $\tan\beta_r$ must be bounded from above. For example, $\tan\beta_r < 9$ in Fig.9 if the experimental lower bound on m_{LH} is taken to be 60 GeV. However, as mentioned before, this upper bound on $\tan\beta_r$ is inconclusive unless the bottom Yukawa coupling is included.

Second, the dependence on the soft SUSY-breaking parameter m_{12} is studied. It is illustrated with a typical example whose parameters are chosen as: $\Lambda = 500$ TeV, $m_1 = 1.6$ TeV, $m_2 = 800$ GeV, m_{top}=180 GeV, and m_{12} is left free. The effect of m_{12} is best seen through m_{LH}, and therefore m_{LH} versus $\frac{m_{12}}{m_2}$ is plotted in Fig.10. As discussed in Section 2, the Higgs sector of $\mathcal{L}_\Lambda$ has an SU(2)×U(1)×U(1) symmetry instead of SU(2)×U(1) in the limit $m_{12} = 0$. Therefore, when $m_{12} = 0$, there is one more U(1) symmetry, and the spontaneous breaking of this additional U(1) symmetry leads to one more massless Goldstone boson, which is exactly the lightest Higgs boson. This explains why $m_{LH} = 0$

in the limit $m_{12} = 0$, and m_{LH} increases linearly with m_{12} (when $\frac{m_{12}}{m_2} < \frac{1}{3}$) in Fig.10. However, when m_{12} is large enough ($\frac{m_{12}}{m_2} > \frac{1}{3}$ for Fig.10), m_{LH} is determined by another mechanism: m_{12} contributes only to the off-diagonal elements of the Higgs squared-mass matrix, and therefore m_{LH} decreases with m_{12} when m_{12} is large enough. In Fig.10, the non-smoothness of the curve around $\frac{m_{12}}{m_2} = \frac{1}{3}$ simply reflects that m_{LH} is determined by different mechanisms for $\frac{m_{12}}{m_2} < \frac{1}{3}$ and $\frac{m_{12}}{m_2} > \frac{1}{3}$. Again, due to the experimental lower bound on m_{LH}, m_{12} must be bounded from both below and above according to Fig.10.

10 Conclusions

In this paper, the idea of Higgs-Higgs condensation is proposed, and it is pointed out how three relevant issues can be solved based on the Higgs-Higgs condensation: the μ problem, the possibility of raising the lightest-Higgs mass, and the triviality problem associated with the Higgs sector. As the first realization of this idea, the Minimal Supersymmetric Model of Higgs-Higgs Condensation is constructed and its qualitative features are studied in detail. Finally, in order to reveal the phenomenological details of the MSMHHC, a systematic study of its parameter space is made. Obviously, an immediate extension of this paper is to include the electroweak SU(2)×U(1) gauge interactions. However, the mere inclusion of gauge interactions may not be enough to solve the fine-tuning problem associated with the soft SUSY-breaking parameters (as discussed in Section 6), and therefore more general structures of the Higgs non-renormalizable interactions should be considered in the future. Another relevant issue that must be faced in the future model-building is that the lack of guiding principles in choosing the structure of Higgs non-renormalizable interactions will undermine the predictive power of the model. As pointed out in Section 6, thanks to the requirement of no fine tuning, this fine-tuning problem actually serves as a guide to the selection of the structure of Higgs non-renormalizable interactions, and therefore both the fine-tuning problem and the issue of the arbitrariness in choosing non-renormalizable interactions may be solved at the same time in future works.

Finally, let's reveal the physical significance of Higgs-Higgs condensation from another viewpoint by asking the following question: "How large can the

lightest-Higgs mass be in supersymmetric theories?" According to the studies of the Non-Minimal SSM [8, 9, 20], the first part of the answer is that the Non-Minimal SSM's are capable of predicting larger lightest-Higgs mass than the MSSM (or the MTCSSM) is. The second part of the answer follows from the present paper: Among the Non-Minimal SSM's, the supersymmetric standard model of Higgs-Higgs condensation is preferred because it is the only choice free from being plagued by the triviality problem. Therefore, if the lightest-Higgs mass predicted by the MSSM is excluded by the future experiments, the supersymmetric model of Higgs-Higgs condensation seems to be the most promising candidate.

Acknowledgement

I would like to thank Professor Mary K. Gaillard for her support and nice advice. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY90-21139.

References

[1] H.P. Nilles, Phys. Rep. **110**, 1 (1984).
J.E. Kim and H.P. Nilles, Phys. Lett. **138B**, 453 (1984).

[2] J.A. Harvey, hep-th/9411097 (1994).

[3] H.E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991).

[4] H.E. Haber and R. Hempfling, SCIPP 91/33 (1992).

[5] *The Higgs Hunter's Guide* by J.F. Gunion et al. (Addison-Wesley, MA, 1990), pp.267-322.

[6] D.J.E. Callaway, Phys. Rep. **167**, 241 (1988).

[7] P. Binétruy and C.A. Savoy, Phys. Lett. **277B**, 453 (1992).

[8] Y.-Y. Wu, Phys. Rev. **D 51**, 5276 (1995).

[9] J. Ellis et al., Phys. Rev. **D 39**, 844 (1989).
M. Drees, Int. J. Mod. Phys. **A4**, 288 (1993).
L.E. Ibanez and J. Mas, Nucl. Phys. **B286**, 107 (1987).
J.P. Derendinger and C. Savoy, Nucl. Phys. **B237**, 307 (1984).
J.M. Frere, D.R.T. Jones and S. Raby, Nucl. Phys. **B222**, 11 (1983).
H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **120B**, 346 (1983).

[10] T.E. Clark, S.T. Love and W.A. Bardeen, Phys. Lett. **237B**, 235 (1990).

[11] M. Carena et al., Nucl. Phys. **B369**, 33 (1992).

[12] P. Binétruy, E.A. Dudas and F. Pillon, Nucl. Phys. **B415**, 175 (1994).

[13] W. Buchmüller and U. Ellwanger, Nucl. Phys. **B245**, 237 (1984).

[14] J. Polchinski and L. Susskind, Phys. Rev. **D 26**, 3661 (1982).

[15] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[16] J.E. Kim and H.P. Nilles, Phys. Lett. **263B**, 79 (1991).

[17] E.J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. **B370**, 105 (1992).

[18] E.J. Chun, hep-ph/9411290 (1994).

[19] G.F. Guidice and A. Masiero, Phys. Lett. **206B**, 480 (1988).
J.A. Casas and C. Munoz, Phys. Lett. **306B**, 288 (1993).
A. Brignole, L.E. Ibanez and C. Munoz, Nucl. Phys. **B422**, 125 (1994).
I. Antoniadis et al., hep-th/9405254 (1994).

[20] T. Moroi and Y. Okada, Phys. Lett. **295B**, 73 (1992).
T. Elliott, S.F. King and P.L. White, Phys. Rev. **D 49**, 2435 (1994).
W.T.A. ter Veldhuis, PURD-TH-92-11 (1992).

[21] M. Lindner, Int. J. Mod. Phys. **A8**, 2167 (1993).

[22] *Supersymmetry and Supergravity* by J. Wess and J. Bagger (Princeton University Press, NJ, 1992).

[23] W. Buchmüller and S.T. Love, Nucl. Phys. **B204**, 213 (1982).

[24] S. Coleman and E. Weinberg, Phys. Rev. **D 7**, 1883 (1973).
R. Barbieri and S. Cecotti, Z. Phys. **C17**, 183 (1983).
P. Binétruy et al., Phys. Rev. **D 37**, 2633 (1988).

[25] N.K. Falck, Z. Phys. **C30**, 247 (1986).

[26] W. Bardeen, Fermilab preprint FERMILAB-CONF-90/269-T (1990).
M. Suzuki, Mod. Phys. Lett. **A5**, 1205 (1990).
A. Hasenfratz et al., Nucl. Phys. **B365**, 79 (1991).

[27] J.L. Lopez et al., Phys. Lett. **306B**, 73 (1993).
Research Directions For The Decade edited by E.L. Berger
(World Scientific, Singapore, 1990).

FIGURE CAPTIONS

Fig.1: The supergraph contributing to the induced kinetic term $\Sigma_N \int d^4\theta\, N^\dagger N[1 + (m_1^2 + m_2^2)\theta^2\bar{\theta}^2]$ for the Higgs chiral superfield N.

Fig.2: A plot of the allowed range of the soft SUSY-breaking parameter B (GeV): $B_{lower} \leq B < B_{upper}$ versus the strength of soft SUSY breaking F_{SUSY} (GeV), where the dotted (solid) line corresponds to B_{upper} (B_{lower}). According to (35), the parameters are chosen as: $\Lambda = 2.5 \times 10^4$ TeV, $f_T = 1.6\,$, $m_1 = 2\,F_{SUSY},\ m_2 = m_T = F_{SUSY},\ m_M = \sqrt{5}\,F_{SUSY},\ m_{12} = \frac{1}{2}\,F_{SUSY}$. B and F_{SUSY} are left free.

Fig.3: Plots of the lightest-Higgs mass m_{LH} (GeV) versus the cut-off Λ (GeV) for two different choices of the top-quark mass: $m_{top} = 164$ GeV and $m_{top} = 180$ GeV. According to (48) and (49), the parameters are chosen as: $F_{SUSY} = 800$ GeV, $m_1 = 2\,F_{SUSY},\ m_2 = F_{SUSY},\ m_{12} = \frac{1}{2}\,F_{SUSY},\ m_{top}$=164 or 180 GeV, and Λ is left free.

Fig.4: Plots of $\tan\beta_r = \frac{v_{2r}}{v_{1r}}$ versus the cut-off Λ (GeV) for two different choices of the top-quark mass: $m_{top} = 164$ GeV and $m_{top} = 180$ GeV. According to (48) and (49), the parameters are chosen as: $F_{SUSY} = 800$ GeV, $m_1 = 2\,F_{SUSY},\ m_2 = F_{SUSY},\ m_{12} = \frac{1}{2}\,F_{SUSY},\ m_{top}$=164 or 180 GeV, and Λ is left free.

Fig.5: Plots of the effective $\mu_r = h_r < \phi_{N_r} >$ (GeV) versus the cut-off Λ (GeV) for two different choices of the top-quark mass: $m_{top} = 164$ GeV and $m_{top} = 180$ GeV. According to (48) and (49), the parameters are chosen as: $F_{SUSY} = 800$ GeV, $m_1 = 2\,F_{SUSY},\ m_2 = F_{SUSY},\ m_{12} = \frac{1}{2}\,F_{SUSY},\ m_{top}$=164 or 180 GeV, and Λ is left free.

Fig.6: A plot of the lightest-Higgs mass m_{LH} (GeV) versus the strength of soft SUSY breaking F_{SUSY} (GeV). According to (48) and (49), the parameters are chosen as: $\Lambda = 500$ TeV, $m_1 = 2\,F_{SUSY}$, $m_2 = F_{SUSY}$, $m_{12} = \frac{1}{2}\,F_{SUSY}$, the top-quark mass m_{top}=180 GeV, and F_{SUSY} is left free.

Fig.7: A plot of the effective $\mu_r = h_r < \phi_{N_r} >$ (GeV) versus the strength of soft SUSY breaking F_{SUSY} (GeV). According to (48) and (49), the parameters are chosen as: $\Lambda = 500$ TeV, $m_1 = 2\,F_{SUSY}$, $m_2 = F_{SUSY}$, $m_{12} = \frac{1}{2}\,F_{SUSY}$, the top-quark mass m_{top}=180 GeV, and F_{SUSY} is left free.

Fig.8: A plot of $\tan\beta_r = \frac{v_{2r}}{v_{1r}}$ versus $\frac{m_1}{m_2}$. According to (48) and (49), the parameters are chosen as: $\Lambda = 500$ TeV, $\sqrt{m_1^2 + m_2^2} = 1$ TeV, $m_{12} = 250$ GeV, the top-quark mass m_{top}=180 GeV, and $\frac{m_1}{m_2}$ is left free.

Fig.9: A plot of the lightest-Higgs mass m_{LH} (GeV) versus $\tan\beta_r = \frac{v_{2r}}{v_{1r}}$. According to (48) and (49), the parameters are chosen as: $\Lambda = 500$ TeV, $\sqrt{m_1^2 + m_2^2} = 1$ TeV, $m_{12} = 250$ GeV, the top-quark mass m_{top}=180 GeV, and $\frac{m_1}{m_2}$ is left free.

Fig.10: A plot of the lightest-Higgs mass m_{LH} (GeV) versus $\frac{m_{12}}{m_2}$. According to (48) and (49), the parameters are chosen as: $\Lambda = 500$ TeV, $m_1 = 1.6$ TeV, $m_2 = 800$ GeV, the top-quark mass m_{top}=180 GeV, and m_{12} is left free.